

**APPARATUS, METHODS, AND COMPUTER PROGRAM PRODUCTS FOR  
REDUCING THE NUMBER OF COMPUTATIONS AND NUMBER OF  
REQUIRED STORED VALUES FOR INFORMATION PROCESSING  
METHODS**

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**CROSS-REFERENCE TO RELATED APPLICATIONS**

The present application claims priority from U.S. provisional patent application S/N 60/210,661, entitled: METHODS AND APPARATUS FOR PROCESSING INFORMATION USING SPECIAL CASE PROCESSING, filed on June 9, 2000, the  
10 contents of which are incorporated herein by reference.

**FIELD OF THE INVENTION**

The present invention relates generally to the determination of coefficients of a function. More particularly, the methods and computer program products of the present  
15 invention relate to reducing the number of computations and values that must be stored in the determination of the coefficients.

**BACKGROUND OF THE INVENTION**

Signal processing is an important function of many electronic systems. In  
20 particular, in many electronic systems, data is transmitted in signal form. Further, some electronic systems analyze and monitor the operation of mechanical or chemical systems by observing the characteristics of signals, such as vibration signals and other types of signals, that are output from these systems. In light of this, methods have been developed to characterize signals such that information or data in the signal is available for data  
25 processing.

As one example, in many electronic systems, time domain signals are typically transformed to the frequency domain prior to signal processing. A typical method for converting signals to the frequency domain is performed using Fourier Transforms. The Fourier Transform of a signal is based on a plurality of samples of the time domain signal  
30 taken over a selected time period, known as the base frequency. Based on these samples of the signal the Fourier Transform provides a plurality of coefficients, where the coefficients respectively represent the amplitude of a frequency that is a multiple of the

base frequency. These coefficients of the Fourier Transform, which represent the signal in the frequency domain, are then used by electronic systems in processing the signal.

Although Fourier Transforms are among some of the most widely used functions for processing signals, there are other functions that are either currently used or will be  
5 used in the future, as a better understanding of their applicability is recognized. These functions include Bessel functions, Legendre Polynomials, Tschhebysheff Polynomials of First and Second Kind, Jacoby Polynomials, Generalized Laguerre Polynomials, Hermite Polynomials, Bernoulli Polynomials, Euler Polynomials, and a variety of Matrices used in Quantum Mechanics, Linear Analysis functions, wavelets and fractals just to name a  
10 few.

Although Fourier transforms and the other functions mentioned above are useful in determining characteristics of signals for use in data processing, there are some drawbacks to their use. Specifically, application of these functions to signals is typically computationally intensive. This is disadvantageous as it may require the use of  
15 specialized processors in order to perform data processing. Further, and even more importantly, the time required to perform the number of computations using these functions may cause an unacceptable delay for many data processing applications. In fact, a goal of many data processing systems is the ability to process data signals in real time, with no delay.

20 For example, the Fourier Series is defined as an infinite series of coefficients representing a signal. To transform a signal using a Fourier Series would require an infinite number of computations. To remedy this problem, many conventional data processing systems use Discrete Fourier Transforms (DFT), as opposed to the infinite Fourier Series. The DFT is the digital approximation to the Fourier Series and is used to  
25 process digitized analog information. Importantly, the DFT replaces the infinite series of the Fourier Series with a finite set of N evenly spaced samples taken over a finite period. The computation of the DFT therefore provides the same number of coefficients as the samples received, instead of an infinite number of samples required by the Fourier Series. As such, use of the DFT provides the most satisfactory current means to process the  
30 signal.

Because of the importance of reducing the time required to process signals, however, methods have been developed to further reduce the number of computations required to perform a DFT of a signal. Specifically, the DFT procedure computes each coefficient by a similar process. The process for a general coefficient is; multiply each  
5 sample by the sine or cosine of the normalized value of the independent variable times the angular rate and sum over all of the samples. This procedure defines N multiply-add steps for each of N coefficients, which in turn, equates to  $N^2$  multiply-add computations per DFT. As many samples of a signal are typically required to perform an adequate approximation of the signal, the DFT of a signal is typically computational and time  
10 intensive.

One of the methods developed to reduce the number of computations is the Butterfly method, which reduces the number of computations from  $N^2$  to N times  $\log(N)$ . The Butterfly method is based on the fact that many of the trigonometric values of the DFT are the same due to periodicity of the functions. As such, the Butterfly method  
15 reduces the matrix associated with the DFT into N/2 two-point transforms (i.e., the transforms representing each coefficient  $a_n$  and  $b_n$ ). The Butterfly method further reduces the redundant trigonometric values of the DFT. Although the Butterfly method reduces the number of computations over the more traditional DFT method, it also adds complexity to the Fourier transformation of a signal. Specifically, the Butterfly method  
20 uses a complex method for addressing the samples of the signal and the matrix containing the functions. This complexity can require the use of specialized processors and increase time for computation of the Fourier Transform. By its nature, the Butterfly is a batch process, which does not begin determination of the coefficients until after all of the samples have been received. Consequently, this method causes latency in the  
25 determination of the coefficients of the function, where the time between the arrival of the last sample and the availability of the coefficients is defined as the latency of the system.

An improved approach to reducing the time required to process signals is described in U.S. Application S/N 09/560,221 entitled: APPARATUS, METHODS,  
30 AND COMPUTER PROGRAM PRODUCTS FOR DETERMINING THE COEFFICIENTS OF A FUNCTION WITH DECREASED LATENCY filed April 28,

2000 and corresponding PCT Application Number WO 00/67146, entitled: Computation of Discrete Fourier Transform, publication date November 9, 2000. These applications are assigned to the inventor of the present application, and are incorporated herein by reference. The approach in WO 00/67146 reduces or eliminates the problem of latency  
5 for processing coefficients by using the property of independence of samples of functions like the DFT. The approach updates at least one of the coefficients of the function prior to receipt of the last sample of a sample set thereby reduce latency.

Despite the improvements in data processing accomplished by the apparatus and methods of U.S. Application 09/560,221), there are continuing needs to reduce the  
10 number of calculations required stored terms for determining the coefficients of a function.

### **SUMMARY OF THE INVENTION**

As set forth below, the apparatus, methods, and computer program products of the present invention overcome many of the deficiencies identified with processing signals  
15 using functions, such as Fourier Transforms. In particular, the present invention provides methods and computer program products that determine the coefficients of a function representative of an input signal with reduced calculation complexity, such that the coefficients of the function are made available within a decreased time from receipt of the last sample of the signal. The present invention also provides methods and computer  
20 program products that reduce the amount calculations that must be performed in order to determine the coefficients of a function, such that less complex hardware designs can be implemented. Specifically, the time for performing computations can be conserved by reusing previously calculated terms so that terms having the same value are calculated fewer than the number of times that the term appears in the original equations; preferably  
25 only one calculation for each repeating term having the same value for the entire calculation process. In addition, some embodiments of the present invention also use a reduced number of values to represent the possible mathematical terms of a function.

### **BRIEF DESCRIPTION OF THE DRAWING AND APPENDICES**

30 Figure is a block diagram illustrating the operations for generating a special case set of equations for converting input values to coefficients with reduced computation

from a more general set of equations according to one embodiment of the present invention.

Appendix 1 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the Fourier Transform of a signal based on eight samples according to one embodiment of the present invention.

Appendix 2 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the Discrete Cosine Transform of a signal based on eight samples according to one embodiment of the present invention.

Appendix 3 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the One-Dimensional Discrete Cosine Transform of a signal based on sixteen samples according to one embodiment of the present invention.

Appendix 4 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the Fourier Transform of a signal based on eight samples according to another embodiment of the present invention.

Appendix 5 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the Fourier Transform of a signal based on eight samples using a different starting from that of Appendix 4 according to another embodiment of the present invention.

Appendix 6 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the Two-Dimensional Discrete Cosine Transform of a signal based on eight sets of eight samples each according to one embodiment of the present invention.

Appendix 7 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the Two-Dimensional Discrete Cosine Transform of a signal based on sixty-four samples each according to one embodiment of the present invention.

Appendix 8 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the Discrete Cosine Transform of a signal based on eight samples according to another embodiment of the present invention.

Appendix 9 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the Fourier Transform of a signal based on eight samples according to another embodiment of the present invention.

Appendix 10 illustrates use of the operations of Figure 1 to reduce the number of calculations for determining the Discrete Cosine Transform of a signal based on eight samples according to another embodiment of the present invention.

Appendix 11 illustrates use of the operations of Figure 1 to both reduce the number of calculations for determining the Fourier Transform and Discrete Cosine Transform of a signal based on sixteen samples according to alternative embodiments of the present invention.

### DETAILED DESCRIPTION OF THE INVENTION

The present invention now will be described more fully hereinafter with reference to the accompanying drawings, in which preferred embodiments of the invention are shown. This invention may, however, be embodied in many different forms and should not be construed as limited to the embodiments set forth herein; rather, these embodiments are provided so that this disclosure will be thorough and complete, and will fully convey the scope of the invention to those skilled in the art. Like numbers refer to like elements throughout.

For illustrative purposes, the various methods and computer program products of the present invention are illustrated and described below in conjunction with the characteristics of Fourier Series. It should be apparent, however, that the methods and computer program products of the present invention can be used with many different types of functions. For instance, the methods and computer program products may be used with functions such as Bessel functions, Legendre Polynomials, Tschebysheff Polynomials of First and Second Kind, Jacoby Polynomials, Generalized Laguerre Polynomials, Hermite Polynomials, Bernoulli Polynomials, Euler Polynomials, and a variety of Matrices used in Quantum Mechanics, Linear Analysis functions, wavelets and fractals. This list is by no means exhaustive and is provided as mere examples. The approach may be applied to any function that can be expressed as a matrix of values. The usefulness of the application of these and other functions not listed above is quite general. The method of the present invention provides a way to develop apparatus, methods, and computer program products for parallel computing and remove calculation redundancy in a rote manner, which is compatible with machine execution. One implementation of the

present invention would be in a general purpose computer program to examine each class of problem and write a minimal execution program or design an apparatus for each specific case of the function. In this application, it would be a programming aid.

In particular, the present invention provides methods and computer program products that remove redundancy from a mathematical procedure for determining coefficients of a function. The present invention is particularly suited to use in systems that employ coefficient-based mathematics.

Some of methods and computer program products of the present invention provide a sequence of operations in which a working formulation or program is examined to identify the functional usage of each part. By classifying the various details of the program and then following rules of combination or substitution that are universal, the number of computations and/or required stored values for a given computational determination may be reduced. As with any algebra, it becomes possible to do significant reconfiguration without knowing the behavior of the actual computational system. The functionality of the entire program is preserved while the process is changed to another form. The form is then algebraically manipulated to optimize speed or whatever parameters are desired.

It is often possible to reduce one or more variables out of a system that has many variables. If only part of the possibilities are addressed at one time this process can be repeated later to handle others. Some of the variables that yield gains when methods according the present invention are applied include the number of bits at various locations, the number of coefficients being formed, and any other aspect that defines the structure.

The number of bits can be used in several ways. One way is simply to detail each of the values of that many bits. Another is to have a continuous function that will be represented in a specific number of bits. If the actual values are to be represented in a small number of bits, it may be that several values of the function will be represented by a single number. In this case, it becomes important to know the correctly rounded points so that no systematic bias will be introduced. Once that has been done correctly, there will be a reduction in the total number of discrete terms and individual computations that must be made available to the process.

The resulting structure is then sorted and processed to algebraically format the output in some basic manner. Typically, the individual formulas of the output terms are related to the primary input terms. Where numbers have replaced the previous functions, the additions are carried out. Once the processing is done, there may be some number of simultaneous equations that share some variables.

The next step is a three-fold set of details to be identified and collected. The first is to identify those variables that are used in identical combinations in different terms of each output function. The second one is to separately determine those variables that are used in identical combinations in various output functions. Finally, the variables should be linked temporally so as to be able to find the time at which storage must be allocated or when all of the members of a group become available. Ordering them temporally in each combinatorial representation can do this. They may be placed in the order of occurrence. It is then easy to access the first member to know when an accumulation site is required. Similarly, it is easy to access the last member to determine the time when all members have become available. The term temporal may be the wrong term when the samples were not from a time-oriented process, but these processed are done on an ordered set of numbers and the order may be artificially be correlated with time.

There may be need to minimize the number of accesses or the number of storage locations or a limit according to specific guidelines. These rules control the next steps to form groups. In the examples supplied, it was necessary to minimize the number of multiplies first. It was then required to minimize the number of add-accumulates. Next, minimize the number of individual storage registers. It is usually beneficial that once a register contains a variable, the variable would not be transferred to other register locations because it allows the registers to be called by the name of the variable it contains and also reduces the number of processing steps defined. After such a variable is used for the last time there may be a newly formed variable that can be assigned to the same register in which case there is value in keeping the register/variable name but changing a prefix or suffix portion. Finally, each output term is to be formed as soon as it becomes determined by the input information. Following this rule is generally beneficial in making the final output available at the earliest time without having to use multi-pass



compiling methods at this level. The procedures do not alter the eventual set of coefficients.

The result is a set of intermediate variables that are each derived from lower variables by a multiply or add process. The actual definition of the intermediates actually contains all of the information required to structure the implementation. The output equations are a sum or product of variables that are defined sequentially descending to the primitives.

Definitions:

10	Alpha (parameters)	Any general formula appearing at a location.
	Beta (parameters)	A numerical value occurring at context/ status.
	T (index)	Token – a numerical value given a symbol.
	Sum (index)	Chunk – a set of input variables summed.
15	Gamma (index)	Term – the product of a chunk and a token.
	C (j)	Coefficient with index j, an output member.
	S (i)	Sample with index i, a member of the ordered input set.

With regard to Figure 1, the basic steps for forming the special case equations is illustrated. Specifically, first the general equations for a set of coefficients are expressed in terms of samples and functions of the x-axis that define the coefficients. (See step 100). The functions are next evaluated to obtain the coefficients in terms of symbolic input numbers and explicit values of the functions. (See step 110). The coefficient equations are then treated as a set of simultaneous equations, (see step 120), and a distinct token is assigned to each different coefficient function value. (See step 130). The symbolic input numbers now associated with each assigned token is sorted within the coefficient equations into groups and assign a new token to each different group. (See step 140). A new token is then assigned to each different group of tokens that is used in the terms. (See step 150). Explicit values of the tokens are then applied to produce a symbolic formula for each coefficient. (See step 160). New token values are then

equations are then rewritten in terms of the token values. (See step 180). After the equations have be rewritten, the apparatus methods and computer program products of the present invention may use the equations to determine the coefficients of a function based on a set of samples.

5           Stated in a somewhat different way, the apparatus, methods, and computer program products of the present invention generate a second set of equations requiring reduced numbers of computations from a first set of general equations, where each general equation defines coefficients in terms of a set of samples and a plurality of functions having respective values. A first set of tokens is initially assigned to the  
10   plurality of functions such that every value of the functions that has a different magnitude is assigned a different token, thereby permitting each general equation to be defined by the set of samples and their associated tokens. Each general equation is then evaluated and the samples having the same associated token are grouped together. A second set of tokens is then assigned to represent a plurality of unique combinations of the samples.  
15   These second set of tokens are then evaluated to determine a third set of tokens that define all the unique tokens of the second set of tokens. This repeated until there are no more redundant tokens. The second set of equations is then generated based on the sets of tokens.

## 20   **Details of the DFT Example**

          It is easiest to view the process as an Output-Oriented Methodology. For the DFT there are N coefficients to be derived from N samples. Each sample is going to be multiplied by a function of the sample position and the specific coefficient. Since a sample has one sample position and will be used in N coefficients, we can designate it by  
25   a generic name and two parameters. The generic name is the name we pick for the initial level of abstraction. Let us pick Alpha. The parameters may be shown as matrix indices. For the current discussion Alpha (i, j) is used to represent the function which evaluates to become the multiplier of the  $i^{\text{th}}$  sample position for the  $j^{\text{th}}$  output coefficient. The indices i and j range from 1 to N. Let i represent the sample positions and j represent the  
30   coefficient being referenced.

For the DFT, the Alpha (i, j) is  $\cos(2\pi i j / N)$  or  $\sin(2\pi i j / N)$  (in here, as well as in the Appendixes, Pi is notation for  $\pi=3.14\dots$ ). Once N is decided, there is a numerical value for each Alpha (i, j). The numerical value will be referred to as Beta (i, j). In the example case there are N times N numerical values designated Beta (i, j).

5        The first operation is to form a list of values. As a designator for the values that we will call tokens, we will use T (m). We begin by setting T (1) equal to Alpha (1,1). We may now proceed in an orderly manner to compare each Alpha (i, j) to T (1). If it is equal we add Alpha (i, j) to the list for T (1), if it is not equal we compare it to T (2) if one has been assigned a value. If it is numerically equal we add it to the list for T (2). If  
10 it does not equal any assigned T-value, we assign it to T (v) where v is the next unassigned index for T. Upon completion, we will have N squared entries. There will be tokens T (1) ... T (v-1). Each Alpha (i, j) may now be replaced by a T (m) whose numerical value is stored in the list and a sign where the Alpha was numerically equal but opposite in value. If at least one of the values of Alpha (i, j) occurred twice, this process  
15 results in a simpler array. As a convention, we will only allow positive values or zero for T (m). All sign information will be left in the equations. Similar simplifications may be done where we check for multiples of Alpha (i, j) or functions of i and j, but in this example we will use only equal amplitudes.

      The second operation is done with the tokens applied. We express each output  
20 function (coefficient C (j),) as the sum of each sample times the token linked with its sample position and the appropriate sign. Each token is multiplied by the sum of all of the samples of the sample positions whose Alpha (i, j) were listed as having the value of that token in the first operation. This operation consists of collecting all of the identical tokens by summing the associated samples into a single term and then multiplying by the  
25 token value. The form of each output function becomes a summation of token products. In this, case j is associated with any one coefficient, so that we are collecting across the i sample positions of each coefficient.

      Let us choose to call the individual samples by a symbol and an index. The  
samples may be denoted S(i). The samples are the actual variables that represent input  
30 information in the final structure. We may then represent the  $j^{\text{th}}$  coefficient by:

$$C(j) = T(1) * (\sum S(dk1)) + T(2) * (\sum S(dk2)) + \dots T(m) * (\sum S(dn)).$$

It follows that the number of multiplies is reduced by each time a token is reused in a coefficient.

Listing the groupings of  $S(i) * T(m)$  that occur across the various coefficients,  $C(j)$ , achieves the next two operations. It is obvious that there are no common groupings of  $S(i)$ s within a coefficient because each sample is only used one time. The  $S(i)$ s are added to form groups. Certain  $S(i)$ s are summed and multiplied by a token. When forming various coefficients the same summation will be used again with another sign or multiplied by another token. It is important to reuse the case where the multiply is by the same token and also the reuse of the summation which will be multiplied by another token and used multiple times.

The next operation is achieved by listing the combinations found in each summation in each coefficient in much the same way as was done in the first operation. It is useful to keep the sample symbols in the order of occurrence within each summation. We will designate the summations by the term chunk. These will be named Sum(1) through Sum(s). A list is formed with the first chunk listed under Sum(1). The second chunk will be compared with it and added to its list if it is identical, else it will be named Sum(2) and so listed. Continue by repeating steps from first operation.

After the coefficients are represented by chunks with signs preserved and times tokens, a list will be made that groups the products of tokens and chunks. These are called terms Gamma(1) through Gamma(t). Terms again are only considered as symbols and signs are preserved as before. Once there is a list of terms which breaks down into a list of chunks, etc. we have the minimum set of elements that are required to produce the set of coefficients.

## 25 **Description of an Example Implementation**

The first constructor is the range of each chunk. Here we want to identify the first and last sample times contained in each. This is where the value of sample order becomes apparent. Storage will be required from the first required sample time. Similarly, the chunk cannot be multiplied by any of the tokens until it is finished. Because the  $S(j)$  are ordered in the chunk definitions listed, no further processing steps are needed at this time.

The second constructor is the sequence of combinations from which to generate the chunks. The general procedure here is to allocate  $N + 1$  storage locations in which to fabricate the final coefficients. It will follow that the first samples can be placed in two locations each and then later samples can be accumulated in the locations. Maintaining the constraint allows the correct solution to be achieved by trial and error. There are geometric and other procedures that also produce the same answer in the case of DFT but they are equivalent and require more words to state. Examples of these procedures are to be seen in the following illustrations. There is no single formula for all types of structures, therefore the trial and error method is a suitable example.

Once the chunks are formed it is possible to produce the terms using the same storage locations but by multiplying and accumulating. The terms are then finally combined into coefficients by add-accumulate procedures. In the more general case, there are other strategies, but these are well-known and would be obvious to anyone skilled in the art.

#### **A Computation Minimization of the Discrete Fourier Transform**

The DFT has been the subject of much development recently. The FFT has been widely accepted as an efficient method of calculation. The FFT requires  $N \cdot \log(N)$  multiplies instead of  $N^2$  for previous methods. The FFT appears to require the least computation of any easily stated paradigm. Embodiments of the present invention provide methods that can be much simpler to execute the FFT. The paradigm uses the least processing power.

An example of a method according to one embodiment of the present invention will now be presented. Although the method of this example can derive a precise minimum computational structure for a certain value of  $N$ , the method does not necessarily specify the structure for another value of  $N$ . This example will show the case where  $N = 8$ . The procedures described can be programmed so that it is possible to mechanically develop the equations for any given  $N$ .

Reference is now made to Appendix 1. Section 1 of Appendix 1 shows the terms that define an 8 by 8 DFT. According to the methods of the standard technologies, each sample along the top must be multiplied by each of the entries below it (i.e. sixty-four

multiplications to be performed), and then the products along each row must be summed to produce the coefficient associated with the row. However, embodiments of the present invention significantly can reduce the number of multiplication steps.

Since the trigonometric functions are actually static in this structure, they have  
5 numerical values. In Section 2, the mathematical evaluations have been performed and the numerical values have been substituted. In each case, the value consists of a sign and a number. These must each be multiplied by the applicable sample value and then summed across the row to form the coefficient. (See steps 100-120).

As can be seen, there are cases where the numerical value of the entry repeats  
10 several times although the sign may reverse, and each sample may have any value in its permitted range. In Section 3, each numerical value other than zero has been assigned a token and the sign has been preserved. (See step 130). Of course, there is no significance to sign in the case of zero.

An algebraic development will now be started. In Section 4, a new column is  
15 marked 'Formula'. The sample numbers are grouped and multiplied by the appropriate tokens. In A0 as an example, the formula is the sum of all of the samples multiplied by the value of token T1. In A3 there is a signed group multiplied by T2 and another by T1. Since all of the variables are either input sample numbers or tokens, this representation is called level zero.

20 Refer to A1 and A3 in Section 4 of Appendix 1. The groups of samples multiplied by T1 differ only in that the signs are all reversed. The same is true of B1 and B3 except that not even the signs are reversed. Although the individual samples are shown here, the significance of each summation is a single number. Later they are named C6 and C4 respectively. If the sum of  $[S1 - S3 - S5 + S7]$  is 3.256, then it follows  
25 that  $A1 = 3.256 * T2 - G5$  and that  $A3 = -3.256 * T2 - G5$ . In other words, we may compute the value of a sum and store it. We can then attach the sign in each case. Once we form the equations for a set of N-coefficients and group sums and tokens, we may identify all groups that are identical (other than sign) independent of where we will use them. The minimum list of groups is one part of developing the required minimum  
30 structure. (See step 140).

The next step will be illustrated graphically, but would be the result of a program in a larger development. In Section 5, there are three levels of summation shown. The variables G1 through G4 each consist of two samples added. These tokens represent unique groups of samples that are multiplied by the first set of tokens T. G1 is the  
5 (N/2)<sup>th</sup> plus the N<sup>th</sup> sample. G2 is sum of the sample before and the sample after the (N/2)<sup>th</sup> sample. G3 is the sum of the two samples immediately outside those of G2. This continues until G(N/2) which is composed of samples S1 and S(N-1). G(N/2 +1) has the same samples as G1 but the second is subtracted from the first. The other N/2 -2 GXs are likewise subtractions and have the same members as G2 through G(N/2-1). The N  
10 variables, GX are level 1, each consisting of the sum or difference of two samples.

The level two CX variables are a third set of tokens that consist of the sum or difference of two level one GX variables. Three pairs are used to make the six CX variables, each are the sum or difference of two GX variables. Therefore, each are sums of four samples and again are just one numerical value and a sign. (See step 150).

15 Level 3 DX variables are a fourth set of tokens that consist of the sum and difference of two CX variables. C1 and C2 are used to form D1 and D2. Therefore, these are each the summation of eight samples reduced to a single signed number. (See steps 160-180).

The method of forming the equations may not be optimized. It is possible to use a  
20 random matching procedure to find the simplest implementation of the required steps. Section 6 contains the same information as Section 4 except the formulas are expressed in variables that represent the actual sample combinations. These combinations are achieved purely by the addition of binary sample numbers in a pattern somewhat like the Butterfly method. It is worth mentioning that they are not identical to the Butterfly  
25 method.

Next, we observe that the value of T1 is one. Multiplying by one produces no change. In Section 7, the formulas are simplified by removing the (times T1) portion. A review of the final formulas shows that there are only four real multiplies to be made. On the other hand, two of these are C4 \* T2 with G7 added to or subtracted from it. The  
30 other two are C6 \* T2 with different signs and G5 subtracted. Therefore, only two

multiplies are required to do an eight-sample ( $N = 8$ ) Fourier Transform (which is  $N/4$ ), the FFT would use 24 which is  $N * \text{Log}(N)$ .

It is necessary to point out that small Fourier Transforms have a higher percentage of 1's and therefore a greater reduction of multiplies. It is estimated that the actual  
5 number of multiplies approaches  $N$  as  $N$  increases. This is for  $N$  being a power of two. Results are not as efficient for other  $N$ 's.

It is also worth noting that for larger  $N$ 's the tokens include  $\frac{1}{2}$  and  $\frac{1}{4}$ . These can be achieved by summing in a bank of memory that includes a wired offset of one or two binary places as it connects to the output bus without multiplying. There are several  
10 ways to take advantage of these cases.

The method of defining level 1 terms seems to be optimal. Formation of the intermediate terms for each  $N$  may be accurately done by a 'modify and evaluate' program that has freedom to substitute equivalents and test the merit of each case. What is best is a matter of the design requirements and the technology being used. Given a  
15 well-chosen set of requirements it is reasonable to devote a sizeable effort to the solution because it is a one-time task. Extension of these principles to  $N = 16, 32$ , and  $64$  are good steps in developing various 'rules' and insights for extending to larger  $N$ 's with less reliance on the 'modify and evaluate' strategy. Once an approach is shown to find the simplest procedure it is possible to phrase various parallel approaches are simply variants  
20 of what is disclosed herein.

A further strategy that is useful when reducing the number of multiplications in a DFT that is composed of a large number of finite-precision samples is achieved by grouping the tokens. This is accomplished as shown in the following example.

An 8192 sample DFT is to be evaluated with samples that are eight-bit binary  
25 numbers. If it is decided to use trigonometric functions, rounded correctly to eight bits. The sixteen-bit products are summed in 32-bit registers. There are 2048 first-level tokens that represent the trigonometric function values. These values are rounded correctly to eight bits and assigned to 256 second-level tokens. All of the components of each coefficient that are multiplied by any of the 2048 first-level tokens that are now  
30 represented by a single second-level token are thus summed and handled in a single multiplication by the appropriate second-level token value.



This has the effect of reducing the number of multiplies and introducing a noise source. A noise source was already present due to quantization of the original samples to eight-bit accuracy. Matching it by eight-bit rounding of the multipliers will increase the noise by the square root of two. If the original noise source was acceptable in the application, general engineering practice considers that there will be no serious degradation due to the use of eight bit multipliers.

It therefore follows that it is possible to further group and sum according to the principles described above using the actual individual properties and bit-lengths that apply to each and every case. This is valuable where a specific case will be used many times such as in real-time processing, modulation or image compression to name a few applications.

### **A Computational Minimization of the Discrete Cosine Transform (DCT)**

The DCT is also of interest for a variety of applications. Appendix 2 Sections 1 to 6 show that the calculation requirements for the Discrete Cosine Transform can be reduced according to embodiments of the present invention.

An example of a method according to one embodiment of the present invention will now be presented for the DCT. This example will show the case where  $N = 8$ . As mentioned earlier, the procedures described here can be programmed so that it is possible to mechanically develop the equations for any given  $N$ .

Appendix 2 Section 1 shows the terms that define an 8 by 8 DCT. Applying essentially the same procedure described in the sequence presented above and illustrated in Figure 1 provides a somewhat different solution as is seen in the listings in Appendix 2. Sections 1 to 6 essentially repeat the previous steps describe for the Fourier Transform in Appendix 1. Again, only a single multiply is required in calculating the coefficients.

Additional descriptions of embodiments of the present invention can be found in Appendices 3-11. An embodiment of the present invention for a sixteen sample one-dimensional discrete cosine transform is presented in Appendix 3. The equations according to an embodiment of the present invention are presented in Appendix 4. The equations in Appendix 4 are derived for an eight sample Fourier transform. Appendix 4 also shows example calculations using those equations. It is to be understood that the

equations, such as those shown in Appendix 4, comprise one embodiment of the present invention, particularly when used in the form of executable steps for processing on an information processors such as a computer. Appendix 4 clearly illustrates an important advantage of the present invention. Specifically, only two multiplication steps are  
5 required in order to calculate the Fourier transform coefficients for the eight input values used in the calculations. Appendix 5 shows calculation similar to those shown in Appendix 4 except that a different starting function is assumed from which coefficients are derived.

Appendix 6 shows an embodiment of the present invention for calculating two-  
10 dimensional discrete cosine transforms for eight set of eight samples. Appendix 7 also shows an embodiment of the present invention for the two-dimensional discrete cosine transform for 64 samples. However, the embodiment shown in Appendix 7 accomplishes the calculations as a single step special case process. Appendix 8 shows an embodiment of present invention for calculating discrete cosine transforms for eight samples.

#### **Process Examples of Fourier Transform and Discrete Cosine Transform**

A simulation program was developed to determine how much storage and the number of adds and multiplies that would be required to process the Fourier Transform (see: Appendix 9 Process Example of SFT) and the Discrete Cosine Transform (see:  
20 Appendix 10 Process Example of SDCT). Since there are eight samples and eight coefficients to be found we can rule out any number below eight. The simulation started with storage elements G1, G2, ... G8. Each of the first four samples was loaded in two locations according to a map of what combinations were needed. The process went as follows: S1: G4, G6; S2: G3, G7; S3: G2, G8; S4: G1, G5. All eight storage elements  
25 thus had information. The next three samples were then add-accumulated to the initial contents as follows: S5: G2, G8; S6: G3, G7; S7: G4, G6.

It is now optional to process or load the eighth sample. The process steps are done by putting the G2 – G4 in a new storage element, TEMP, G2 + G4 in G4, and then TEMP in G2. It is then possible to put G6 + G8 in TEMP, G6 – G8 in G6 and then  
30 TEMP in G8. B2 is now in G6.

The next step is to multiply G8 times T2 ( $T2 = 0.7071067812\dots$ ) and place  $G8 \cdot T2$  in TEMP. Then place  $TEMP - G7$  in G8 and  $TEMP + G7$  in G7. B1 is now in G7 and B3 is in G8. We therefore see that it is possible to compute all of the  $B_n$  after the  $(N - 1)$  sample as they are independent of the  $N^{th}$  sample.

5        The process has used 16 add-accumulate procedures and one multiply. A0 is now in G4, A2 is in G1 and A4 is in G3.

The other coefficient requires that G2 is multiplied by T2. First place  $G2 \cdot T2$  in TEMP. Now place  $G5 - TEMP$  in G2 and  $G5 + TEMP$  in G5. Now A1 is in G2 and A3 is in G5.

10       The coefficients are complete and the entire process used a total of twenty add-accumulate procedures, two multiplications. The Butterfly method by comparison uses 24 multiply-add-accumulate procedures.

The total storage requirement is  $N + 1$  words. In a practical situation there is usually a requirement to be saving one sample set while processing the next. It follows  
15       that some amount of storage (up to  $N$  words) will be added to store the samples that would be received before the coefficients are transmitted.

Embodiments of the present invention are of value when the original program is dedicated to a specific purpose such as evaluating the actual values of functions at selected points of evaluation. Once this information is determined, the entire solution is  
20       formed with the real variables being unrestricted. The program is then structured for optimum behavior and made ready for the input values to be applied with no difference or controlled changes in the functional behavior. The process can improve the understandability of mathematical relationships and identify cases where variables change their effects.

25       In general, the process of generating coefficient from input values is parallel to the process of generating input values from coefficients. Therefore, the methods shown to do the former are equally methods to do the latter.

Appendices 1- 11 are method steps, tables, examples, and control flow illustrations of methods and program products according to the invention. It will be  
30       understood that each step, flowchart and control flow illustrations, and combinations thereof can be implemented by computer program instructions. These computer program

instructions may be loaded onto a computer or other programmable apparatus to produce a machine, such that the instructions which execute on the computer or other programmable apparatus create means for implementing the functions specified in the block diagram, flowchart or control flow block(s) or step(s). These computer program instructions may also be stored in a computer-readable memory that can direct a computer or other programmable apparatus to function in a particular manner, such that the instructions stored in the computer-readable memory produce an article of manufacture including instruction means which implement the function specified in the block diagram, flowchart or control flow block(s) or step(s). The computer program instructions may also be loaded onto a computer or other programmable apparatus to cause a series of operational steps to be performed on the computer or other programmable apparatus to produce a computer-implemented process such that the instructions which execute on the computer or other programmable apparatus provide steps for implementing the functions specified in the block diagram, flowchart or control flow block(s) or step(s).

Accordingly, blocks or steps of the block diagram, flowchart or control flow illustrations support combinations of means for performing the specified functions, combinations of steps for performing the specified functions and program instruction means for performing the specified functions. It will also be understood that each block or step of the block diagram, flowchart or control flow illustrations, and combinations of blocks or steps in the block diagram, flowchart or control flow illustrations, can be implemented by special purpose hardware-based computer systems which perform the specified functions or steps, or combinations of special purpose hardware and computer instructions.

Many modifications and other embodiments of the invention will come to mind to one skilled in the art to which this invention pertains, having the benefit of the teachings presented in the foregoing descriptions and the associated drawings. Therefore, it is to be understood that the invention is not to be limited to the specific embodiments disclosed and that modifications and other embodiments are intended to be included within the scope of the appended claims. Although specific terms are employed herein, they are used in a generic and descriptive sense only and not for purposes of limitation.

### Appendix 1

Example of steps for reducing the calculations for determining the Fourier Transform for eight samples according to one embodiment of the present invention.

Co	Sampl 1	Sampl 2	Sampl 3	Sampl 4	Sampl 5	Sampl 6	Sampl 7	Sampl 8
Section 1								
A0	cos(0)	cos(0)	cos(0)	cos(0)	cos(0)	cos(0)	cos(0)	cos(0)
A1	cos(Pi/4)	cos(Pi/2)	cos(Pi*3/4)	cos(Pi)	cos(Pi*5/4)	cos(Pi*3/2)	cos(Pi*7/4)	cos(2*Pi)
A2	cos(Pi/2)	cos(Pi)	cos(Pi*3/2)	cos(2*Pi)	cos(Pi*5/2)	cos(Pi*6/2)	cos(Pi*7/2)	cos(4*Pi)
A3	cos(Pi*3/4)	cos(Pi*3/2)	cos(Pi*9/4)	cos(3*Pi)	cos(Pi*15/4)	cos(Pi*9/2)	cos(Pi*21/4)	cos(6*Pi)
A4	cos(Pi)	cos(2*Pi)	cos(3*Pi)	cos(4*Pi)	cos(5*Pi)	cos(6*Pi)	cos(7*Pi)	cos(8*Pi)
B1	sin(Pi/4)	sin(Pi/2)	sin(Pi*3/4)	sin(Pi)	sin(Pi*5/4)	sin(Pi*3/2)	sin(Pi*7/4)	sin(2*Pi)
B2	sin(Pi/2)	sin(Pi)	sin(Pi*3/2)	sin(2*Pi)	sin(Pi*5/2)	sin(Pi*6/2)	sin(Pi*7/2)	sin(4*Pi)
B3	sin(Pi*3/4)	sin(Pi*3/2)	sin(Pi*9/4)	sin(3*Pi)	sin(Pi*15/4)	sin(Pi*9/2)	sin(Pi*21/4)	sin(6*Pi)
Section 2								
A0	+1	+1	+1	+1	+1	+1	+1	+1
A1	+0.707107	0	-0.707107	-1	-0.707107	0	+0.707107	+1
A2	0	-1	0	+1	0	-1	0	+1
A3	-0.707107	0	+0.707107	-1	+0.707107	0	-0.707107	+1
A4	-1	+1	-1	+1	-1	+1	-1	+1
B1	+0.707107	+1	+0.707107	0	-0.707107	-1	-0.707107	0
B2	+1	0	-1	0	+1	0	-1	0
B3	+0.707107	-1	+0.707107	0	-0.707107	1	-0.707107	0
Section 3								
A0	T1	T1	T1	T1	T1	T1	T1	T1
A1	T2	0	-T2	-T1	-T2	0	T2	T1
A2	0	-T1	0	T1	0	-T1	0	T1
A3	-T2	0	T2	-T1	T2	0	-T2	T1
A4	-T1	T1	-T1	T1	-T1	T1	-T1	T1
B1	T2	T1	T2	0	-T2	-T1	-T2	0
B2	T1	0	-T1	0	T1	0	-T1	0
B3	T2	-T1	T2	0	-T2	T1	-T2	0

STATUS	Name	Formula	S1	S2	S3	S4	S5	S6	S7	S8	NOTES
Section 4											
Level 0	A0	$[S1+S2+S3+S4+S5+S6+S7+S8]*T1$	T1	T1	T1	T1	T1	T1	T1	T1	
Level 0	A1	$[S1-S3-S5+S7]*T2 + [-S4+S8]*T1$	T2	0	-T2	-T1	-T2	0	T2	T1	
Level 0	A2	$[-S2+S4-S6+S8]*T1$	0	-T1	0	T1	0	-T1	0	T1	
Level 0	A3	$[-S1+S3+S5-S7]*T2 + [-S4+S8]*T1$	-T2	0	T2	-T1	T2	0	-T2	T1	
Level 0	A4	$[-S1+S2-S3+S4-S5+S6-S7+S8]*T1$	-T1	T1	-T1	T1	-T1	T1	-T1	T1	
Level 0	B1	$[S1+S3-S5-S7]*T2 + [S2-S6]*T1$	T2	T1	T2	0	-T2	-T1	-T2	0	**
Level 0	B2	$[S1-S3+S5-S7]*T1$	T1	0	-T1	0	T1	0	-T1	0	**
Level 0	B3	$[S1+S3-S5-S7]*T2 + [-S2+S6]*T1$	T2	-T1	T2	0	-T2	T1	-T2	0	**
Section 5											
Level 1	G1	$S4+S8$				+				+	
Level 1	G2	$S3+S5$			+		+				
Level 1	G3	$S2+S6$		+				+			
Level 1	G4	$S1+S7$	+						+		
Level 1	G5	$S4-S8$	+			+				-	***
Level 1	G6	$S1-S7$	+								***
Level 1	G7	$S2-S6$		+							***
Level 1	G8	$S3-S5$			+						
Level 2	C1	$G1+G3$	+	+		+		+		+	
Level 2	C2	$G2+G4$	+		+		+		+		
Level 2	C3	$G6-G8$	+		-		+		-		***
Level 2	C4	$G6+G8$	+		+		+				***
Level 2	C5	$G1-G3$	-	-		+				+	***
Level 2	C6	$G2-G4$			+		+				***
Level 3	D1	$C1+C2$	+	+	+	+	+	+	+	+	***
Level 3	D2	$C1-C2$	-	+	-	+	-	+	-	+	***

\*\* = Complete on arrival of the (N-1)<sup>th</sup> sample. \*\*\* = Term is used in the final Formula set.

STATUS	Name	Formula	S1	S2	S3	S4	S5	S6	S7	S8	FINAL
Section 6											
simplified	A0	D1*T1	T1	T1	T1	T1	T1	T1	T1	T1	
simplified	A1	-C6*T2-G5*T1	T2	0	-T2	-T1	-T2	0	T2	T1	
simplified	A2	C5*T1	0	-T1	0	T1	0	-T1	0	T1	
simplified	A3	C6*T2-G5*T1	-T2	0	T2	-T1	T2	0	-T2	T1	
simplified	A4	D2*T1	-T1	T1	-T1	T1	-T1	T1	-T1	T1	
simplified	B1	C4*T2+G7*T1	T2	T1	T2	0	-T2	-T1	-T2	0	**
simplified	B2	C3*T1	T1	0	-T1	0	T1	0	-T1	0	**
simplified	B3	C4*T2-G7*T1	T2	-T1	T2	0	-T2	T1	-T2	0	**
Section 7											
Final	A0	D1	1	1	1	1	1	1	1	1	
Final	A1	-C6*T2-G5	T2	0	-T2	-1	-T2	0	T2	1	
Final	A2	C5	0	-1	0	1	0	-1	0	1	
Final	A3	C6*T2-G5	-T2	0	T2	-1	T2	0	-T2	1	
Final	A4	D2	-1	1	-1	1	-1	1	-1	1	
Final	B1	C4*T2+G7	T2	1	T2	0	-T2	-1	-T2	0	**
Final	B2	C3	1	0	-1	0	1	0	-1	0	**
Final	B3	C4*T2-G7	T2	-1	T2	0	-T2	1	-T2	0	**

## Appendix 2

Example of steps for reducing the calculations for determining the Discrete Cosine Transform for 8 samples according to one embodiment of the present invention. It is known that symmetries in the DCT lead to a number of identical coefficients due to the use of the cosine function ( $\cos(a) = \cos(2\pi - a)$ ). The table below illustrates the full description for completeness, yet in actual computations the symmetry leads to the fact that  $A7=A1$ ,  $A6=A2$ ,  $A5=A3$ .

Co	Sampl 1	Sampl 2	Sampl 3	Sampl 4	Sampl 5	Sampl 6	Sampl 7	Sampl 8
Section 1								
A0	$\cos(0)$	$\cos(0)$	$\cos(0)$	$\cos(0)$	$\cos(0)$	$\cos(0)$	$\cos(0)$	$\cos(0)$
A1	$\cos(\pi/4)$	$\cos(\pi/2)$	$\cos(3\pi/4)$	$\cos(\pi)$	$\cos(5\pi/4)$	$\cos(3\pi/2)$	$\cos(7\pi/4)$	$\cos(2\pi)$
A2	$\cos(\pi/2)$	$\cos(\pi)$	$\cos(3\pi/2)$	$\cos(2\pi)$	$\cos(5\pi/2)$	$\cos(6\pi/2)$	$\cos(7\pi/2)$	$\cos(4\pi)$
A3	$\cos(3\pi/4)$	$\cos(3\pi/2)$	$\cos(9\pi/4)$	$\cos(3\pi)$	$\cos(15\pi/4)$	$\cos(9\pi/2)$	$\cos(21\pi/4)$	$\cos(6\pi)$
A4	$\cos(\pi)$	$\cos(2\pi)$	$\cos(3\pi)$	$\cos(4\pi)$	$\cos(5\pi)$	$\cos(6\pi)$	$\cos(7\pi)$	$\cos(8\pi)$
A5	$\cos(5\pi/4)$	$\cos(5\pi/2)$	$\cos(15\pi/4)$	$\cos(5\pi)$	$\cos(25\pi/4)$	$\cos(15\pi/2)$	$\cos(35\pi/4)$	$\cos(10\pi)$
A6	$\cos(3\pi/2)$	$\cos(3\pi)$	$\cos(9\pi/2)$	$\cos(6\pi)$	$\cos(15\pi/2)$	$\cos(9\pi)$	$\cos(21\pi/2)$	$\cos(12\pi)$
A7	$\cos(7\pi/4)$	$\cos(7\pi/2)$	$\cos(21\pi/4)$	$\cos(7\pi)$	$\cos(35\pi/4)$	$\cos(21\pi/2)$	$\cos(49\pi/4)$	$\cos(14\pi)$
Section 2								
A0	+1	+1	+1	+1	+1	+1	+1	+1
A1	+0.707107	0	-0.707107	-1	-0.707107	0	+0.707107	+1
A2	0	-1	0	+1	0	-1	0	+1
A3	-0.707107	0	+0.707107	-1	+0.707107	0	-0.707107	+1
A4	-1	+1	-1	+1	-1	+1	-1	+1
A5	-0.707107	0	+0.707107	-1	+0.707107	0	-0.707107	+1
A6	0	-1	0	+1	0	-1	0	+1
A7	+0.707107	0	-0.707107	-1	-0.707107	0	+0.707107	+1
Section 3								
A0	T1	T1	T1	T1	T1	T1	T1	T1
A1	T2	0	-T2	-T1	-T2	0	T2	T1
A2	0	-T1	0	T1	0	-T1	0	T1
A3	-T2	0	T2	-T1	T2	0	-T2	T1
A4	-T1	T1	-T1	T1	-T1	T1	-T1	T1
A5	-T2	0	T2	-T1	+T2	0	-T2	T1



# Exhibit 23

Co	Sampl 1	Sampl 2	Sampl 3	Sampl 4	Sampl 5	Sampl 6	Sampl 7	Sampl 8
A6	0	-T1	0	T1	0	-T1	0	T1
A7	T2	0	-T2	-T1	-T2	0	T2	T1

Raw Formula	Name	USE	S1	S2	S3	S4	S5	S6	S7	S8	FINAL
Section 4											
[S1+S2+S3+S4+S5+S6+S7+S8]*T1	A0	Coef.	T1	T1	T1	T1	T1	T1	T1	T1	
[S1-S3-S5+S7]*T2+[-S4+S8]T1	A1	Coef.	T2	0	-T2	-T1	-T2	0	T2	T1	
[-S2+S4-S6+S8]*T1	A2	Coef.	0	-T1	0	T1	0	-T1	0	T1	
[-S1+S3+S5-S7]*T2+[-S4+S8]*T1	A3	Coef.	-T2	0	T2	-T1	T2	0	-T2	T1	
[-S1+S2-S3+S4-S5+S6-S7+S8]*T1	A4	Coef.	-T1	T1	-T1	T1	-T1	T1	-T1	T1	
[-S1+S3+S5-S7]*T2+[-S4+S8]*T1	A5	Coef.	-T2	0	T2	-T1	T2	0	-T2	T1	
[-S2+S4-S6+S8]*T1	A6	Coef.	0	-T1	0	T1	0	-T1	0	T1	
[S1-S3-S5+S7]*T2+[-S4+S8]*T1	A7	Coef.	T2	0	-T2	-T1	-T2	0	T2	T1	
Section 5											
S4+S8	G1	Level 1				+				+	
S3+S5	G2	Level 1			+		+				
S2+S6	G3	Level 1		+				+			
S1+S7	G4	Level 1	+						+		
S4-S8	G5	Level 1				+				-	G5
G1+G3	C1	Level 2		+		+		+		+	
G2+G4	C2	Level 2	+		+		+		+		
G1-G3	C5	Level 2		-		+		-		+	C5
G2-G4	C6	Level 2	-		+		+		-		C6
C1+C2	D1	Level 3	+	+	+	+	+	+	+	+	D1
C1-C2	D2	Level 3	-	+	-	+	-	+	-	+	D2
Section 6 (T1 = 1, T2 = 0.707107...)											
D1	A0		+1	+1	+1	+1	+1	+1	+1	+1	
-C6*T2-G5	A1		+7		-7	-1	-7		+7	+1	
C5	A2			-1		+1		-1		+1	
C6*T2-G5	A3		-7		+7	-1	+7		-7	+1	
D2	A4		-1	+1	-1	+1	-1	+1	-1	+1	
C6*T2-G5	A5		-7		+7	-1	+7		-7	+1	
C5	A6			-1		+1		-1		+1	
-C6*T2-G5	A7		+7		-7	-1	-7		+7	+1	

### Appendix 3

Example of steps for reducing the calculations for determining the One Dimensional Discrete Cosine Transform for 16 samples according to one embodiment of the present invention.

Co	Formula	T2A	T2B	T2C	T2D	T3A	T3B	T3C	T3D	T4A	T4B	T4C	T4D	T5A	T5B	T5C	T5D
A1	S1+S15																
A2	S2+S14																
A3	S3+S13																
A4	S4+S12																
A5	S5+S11																
A6	S6+S10																
A7	S7+S9																
A8	S8+S16																
A9	S8-S16																
B1	A1+A7																
B2	A2+A6																
B3	A3+A5																
B4	A4+A8																
B5	A1-A7																
B6	A2-A6																
B7	A3-A5																
B8	A4-A8																
C1	B1+B3																
C2	B2+B4																
C3	B1-B3																
C4	B2-B4																
D1	C1+C2																
D2	C1-C2																
T1	0																
T2	1																
T3	0.9238795325																
T4	0.7071067814																
T5	0.3826834324																
M1	B5*T3																
M2	B5*T5																
M3	B7*T3																
M4	B7*T5																
M5	B6*T4																
M6	C3*T4																
E1	-A9+M5																

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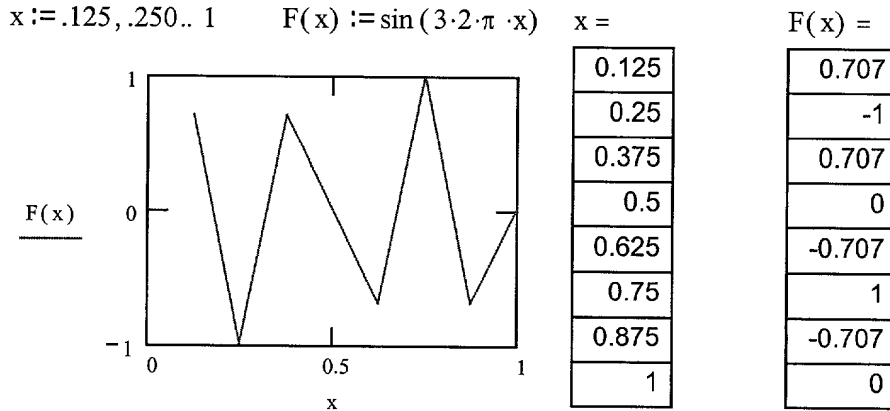
Co	Formula	T2A	T2B	T2C	T2D	T3A	T3B	T3C	T3D	T4A	T4B	T4C	T4D	T5A	T5B	T5C	T5D
E2	-A9-M5																
E3	M1+M4																
E4	M2-M3																
E5	-M6-B8																
E6	M6-B8																
Co1	D1				D1												
Co2	E1+E3	-A9					M1				M5				M4		
Co3	E5		-B8									-M6					
Co4	E2+E4	-A9					-M3				-M5				M2		
Co5	-C4			-C4													
Co6	E2-E4	-A9					M3				-M5				-M2		
Co7	E6		-B8									M6					
Co15	E5		-B8									-M6					
Co16	E1+E3	-A9					M1				M5				M4		
Co8	E1-E3	-A9					-M1				M5				-M4		
Co9	-D2				-D2												
Co10	E1-E3	-A9					-M1				M5				-M4		
Co11	E6		-B8									M6					
Co12	E2-E4	-A9					M3				-M5				-M2		
Co13	-C4			-C4													
Co14	E2+E4	-A9					-M3				-M5				M2		

## Appendix 4

Example calculations of Fourier Transform for eight samples according to one embodiment of the present invention.

### Super Fast Fourier Transform

Let x range from 1/8 to 1 in 8 steps. Let F(x) be sampled at the eight points.



$$S1 := F(.125) \quad S2 := F(.250) \quad S3 := F(.375) \quad S4 := F(.5)$$

$$S5 := F(.625) \quad S6 := F(.75) \quad S7 := F(.875) \quad S8 := F(1)$$

$$T2 := \sin\left(\frac{\pi}{4}\right)$$

$$M1 := (S1 - S3 - S5 + S7) \cdot T2 \quad \text{Multiply \#1}$$

$$M2 := (S1 + S3 - S5 - S7) \cdot T2 \quad \text{Multiply \#2}$$

$$A0 := (S1 + S2 + S3 + S4 + S5 + S6 + S7 + S8) \quad \frac{A0}{2} = 0$$

$$A1 := M1 - S4 + S8 \quad A1 = 0$$

$$A2 := (-S2 + S4 - S6 + S8) \quad A2 = 0$$

$$A3 := -M1 - S4 + S8 \quad A3 = 0$$

$$A4 := -S1 + S2 - S3 + S4 - S5 + S6 - S7 + S8 \quad \frac{A4}{2} = 0$$

$$B1 := M2 + S2 - S6 \quad B1 = 0$$

$$B2 := S1 - S3 + S5 - S7 \quad B2 = 0$$

$$B3 := M2 - S2 + S6 \quad B3 = 4$$

$$FT := \begin{bmatrix} A1 & A2 & A3 & \frac{A4}{2} \\ B1 & B2 & B3 & \frac{A0}{2} \end{bmatrix}$$

$$FT = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix}$$

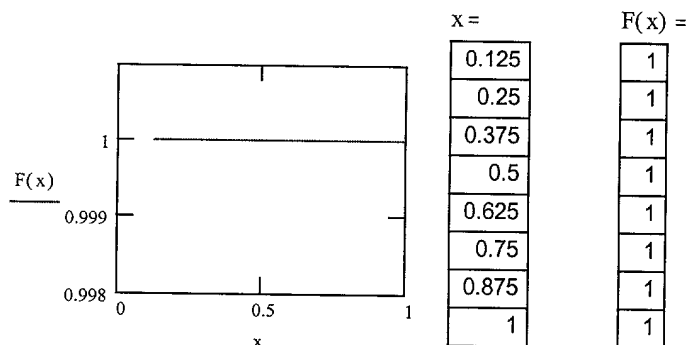
## Appendix 5

Example calculations of Fourier Transform for eight samples according to one embodiment of the present invention.

### Super Fast Fourier Transform

Let  $x$  range from  $1/8$  to  $1$  in 8 steps. Let  $F(x)$  be sampled at the eight points.

$$x := .125, .250, \dots, 1 \quad F(x) := \cos(0.2 \cdot \pi \cdot x)$$



$$S1 := F(.125) \quad S2 := F(.250) \quad S3 := F(.375) \quad S4 := F(.5)$$

$$S5 := F(.625) \quad S6 := F(.75) \quad S7 := F(.875) \quad S8 := F(1)$$

$$T2 := \sin\left(\frac{\pi}{4}\right)$$

$$M1 := (S1 - S3 - S5 + S7) \cdot T2 \quad \text{Multiply \#1}$$

$$M2 := (S1 + S3 - S5 - S7) \cdot T2 \quad \text{Multiply \#2}$$

$$A0 := (S1 + S2 + S3 + S4 + S5 + S6 + S7 + S8) \quad \frac{A0}{2} = 4$$

$$A1 := M1 - S4 + S8 \quad A1 = 0$$

$$A2 := (-S2 + S4 - S6 + S8) \quad A2 = 0$$

$$A3 := -M1 - S4 + S8 \quad A3 = 0$$

$$A4 := -S1 + S2 - S3 + S4 - S5 + S6 - S7 + S8 \quad \frac{A4}{2} = 0$$

$$B1 := M2 + S2 - S6 \quad B1 = 0$$

$$B2 := S1 - S3 + S5 - S7 \quad B2 = 0$$

$$B3 := M2 - S2 + S6 \quad B3 = 0$$

$$FT := \begin{bmatrix} A1 & A2 & A3 & \frac{A4}{2} \\ B1 & B2 & B3 & \frac{A0}{2} \end{bmatrix} \quad FT = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

## Appendix 6

Example calculations of 2-Dimension Discrete Cosine Transform for eight sets of eight samples according to one embodiment of the present invention.

### Super Fast 2-Dimension Discrete Cosine Transform

Let  $F(x,y)$  be any real single-valued function.  $[S]$  will form an  $8 \times 8$  matrix which will then be used as input. The values thus obtained are displayed below and to the right of the initial equations. Eight sets of eight coefficients are evaluated as  $[A]$ . After  $[A]$  is formed, the process is repeated sideways to form the two-dimension discrete cosine transform. One multiply is required for each eight sample, one dimension transform. A total of sixteen multiplies is therefore necessary to complete the process.

$$F(x,y) := \cos\left(2 \cdot \pi \cdot 4 \cdot y\right) + \frac{x}{3} - \frac{y}{5}$$

$$T1 := 1$$

$$T2 := 0.7071067812$$

$$S := \begin{pmatrix} F(.125,.125) & F(.25,.125) & F(.375,.125) & F(.5,.125) & F(.625,.125) & F(.75,.125) & F(.875,.125) & F(1,.125) \\ F(.125,.250) & F(.25,.250) & F(.375,.250) & F(.5,.250) & F(.625,.250) & F(.75,.250) & F(.875,.250) & F(1,.250) \\ F(.125,.375) & F(.25,.375) & F(.375,.375) & F(.5,.375) & F(.625,.375) & F(.75,.375) & F(.875,.375) & F(1,.375) \\ F(.125,.5) & F(.25,.5) & F(.375,.5) & F(.5,.5) & F(.625,.5) & F(.75,.5) & F(.875,.5) & F(1,.5) \\ F(.125,.625) & F(.25,.625) & F(.375,.625) & F(.5,.625) & F(.625,.625) & F(.75,.625) & F(.875,.625) & F(1,.625) \\ F(.125,.75) & F(.25,.75) & F(.375,.75) & F(.5,.75) & F(.625,.75) & F(.75,.75) & F(.875,.75) & F(1,.75) \\ F(.125,.875) & F(.25,.875) & F(.375,.875) & F(.5,.875) & F(.625,.875) & F(.75,.875) & F(.875,.875) & F(1,.875) \\ F(.125,1) & F(.25,1) & F(.375,1) & F(.5,1) & F(.625,1) & F(.75,1) & F(.875,1) & F(1,1) \end{pmatrix}$$

$$S = \begin{pmatrix} -0.983 & -0.942 & -0.9 & -0.858 & -0.817 & -0.775 & -0.733 & -0.692 \\ 0.992 & 1.033 & 1.075 & 1.117 & 1.158 & 1.2 & 1.242 & 1.283 \\ -1.033 & -0.992 & -0.95 & -0.908 & -0.867 & -0.825 & -0.783 & -0.742 \\ 0.942 & 0.983 & 1.025 & 1.067 & 1.108 & 1.15 & 1.192 & 1.233 \\ -1.083 & -1.042 & -1 & -0.958 & -0.917 & -0.875 & -0.833 & -0.792 \\ 0.892 & 0.933 & 0.975 & 1.017 & 1.058 & 1.1 & 1.142 & 1.183 \\ -1.133 & -1.092 & -1.05 & -1.008 & -0.967 & -0.925 & -0.883 & -0.842 \\ 0.842 & 0.883 & 0.925 & 0.967 & 1.008 & 1.05 & 1.092 & 1.133 \end{pmatrix}$$

$$G10 := S_{0,3} + S_{0,7}$$

$$G20 := S_{0,2} + S_{0,4}$$

$$G30 := S_{0,1} + S_{0,5}$$

$$G40 := S_{0,0} + S_{0,6}$$

$$G50 := S_{0,3} - S_{0,7}$$

$$C10 := G10 + G30$$

$$C20 := G20 + G40$$

$$C50 := G10 - G30$$

$$C60 := G20 - G40$$

$$D10 := C10 + C20$$

$$D20 := C10 - C20$$

$$G11 := S_{1,3} + S_{1,7}$$

$$G21 := S_{1,2} + S_{1,4}$$

$$G31 := S_{1,1} + S_{1,5}$$

$$G41 := S_{1,0} + S_{1,6}$$

$$G51 := S_{1,3} - S_{1,7}$$

$$C11 := G11 + G31$$

$$C21 := G21 + G41$$

$$C51 := G11 - G31$$

$$C61 := G21 - G41$$

$$D11 := C11 + C21$$

$$D21 := C11 - C21$$

$$G12 := S_{2,3} + S_{2,7}$$

$$G22 := S_{2,2} + S_{2,4}$$

$$G32 := S_{2,1} + S_{2,5}$$

$$G42 := S_{2,0} + S_{2,6}$$

$$G52 := S_{2,3} - S_{2,7}$$

$$C12 := G12 + G32$$

$$C22 := G22 + G42$$

$$C52 := G12 - G32$$

$$C62 := G22 - G42$$

$$D12 := C12 + C22$$

$$D22 := C12 - C22$$

G13:= S <sub>3,3</sub> + S <sub>3,7</sub>	G33:= S <sub>3,1</sub> + S <sub>3,5</sub>	G53:= S <sub>3,3</sub> - S <sub>3,7</sub>
G23:= S <sub>3,2</sub> + S <sub>3,4</sub>	G43:= S <sub>3,0</sub> + S <sub>3,6</sub>	C13:= G13+ G33
C23:= G23+ G43	C53:= G13- G33	C63:= G23- G43
D13:= C13+ C23	D23:= C13- C23	
G14:= S <sub>4,3</sub> + S <sub>4,7</sub>	G34:= S <sub>4,1</sub> + S <sub>4,5</sub>	G54:= S <sub>4,3</sub> - S <sub>4,7</sub>
G24:= S <sub>4,2</sub> + S <sub>4,4</sub>	G44:= S <sub>4,0</sub> + S <sub>4,6</sub>	C14:= G14+ G34
C24:= G24+ G44	C54:= G14- G34	C64:= G24- G44
D14:= C14+ C24	D24:= C14- C24	
G15:= S <sub>5,3</sub> + S <sub>5,7</sub>	G35:= S <sub>5,1</sub> + S <sub>5,5</sub>	G55:= S <sub>5,3</sub> - S <sub>5,7</sub>
G25:= S <sub>5,2</sub> + S <sub>5,4</sub>	G45:= S <sub>5,0</sub> + S <sub>5,6</sub>	C15:= G15+ G35
C25:= G25+ G45	C55:= G15- G35	C65:= G25- G45
D15:= C15+ C25	D25:= C15- C25	
G16:= S <sub>6,3</sub> + S <sub>6,7</sub>	G36:= S <sub>6,1</sub> + S <sub>6,5</sub>	G56:= S <sub>6,3</sub> - S <sub>6,7</sub>
G26:= S <sub>6,2</sub> + S <sub>6,4</sub>	G46:= S <sub>6,0</sub> + S <sub>6,6</sub>	C16:= G16+ G36
C26:= G26+ G46	C56:= G16- G36	C66:= G26- G46
D16:= C16+ C26	D26:= C16- C26	
G17:= S <sub>7,3</sub> + S <sub>7,7</sub>	G37:= S <sub>7,1</sub> + S <sub>7,5</sub>	G57:= S <sub>7,3</sub> - S <sub>7,7</sub>
G27:= S <sub>7,2</sub> + S <sub>7,4</sub>	G47:= S <sub>7,0</sub> + S <sub>7,6</sub>	C17:= G17+ G37
C27:= G27+ G47	C57:= G17- G37	C67:= G27- G47
D17:= C17+ C27	D27:= C17- C27	

M0 := C60 T2	M1 := C61 T2	M2 := C62 T2	M3 := C63 T2
A00 := D10	A10 := D11	A20 := D12	A30 := D13
A01 := -M0 - G50	A11 := -M1 - G51	A21 := -M2 - G52	A31 := -M3 - G53
A02 := C50	A12 := C51	A22 := C52	A32 := C53
A03 := M0 - G50	A13 := M1 - G51	A23 := M2 - G52	A33 := M3 - G53
A04 := D20	A14 := D21	A24 := D22	A34 := D23
A05 := M0 - G50	A15 := M1 - G51	A25 := M2 - G52	A35 := M3 - G53
A06 := C50	A16 := C51	A26 := C52	A36 := C53
A07 := -M0 - G50	A17 := -M1 - G51	A27 := -M2 - G52	A37 := -M3 - G53

M4 := C64 T2	M5 := C65 T2	M6 := C66 T2	M7 := C67 T2
A40 := D14	A50 := D15	A60 := D16	A70 := D17
A41 := -M4 - G54	A51 := -M5 - G55	A61 := -M6 - G56	A71 := -M7 - G57
A42 := C54	A52 := C55	A62 := C56	A72 := C57
A43 := M4 - G54	A53 := M5 - G55	A63 := M6 - G56	A73 := M7 - G57
A44 := D24	A54 := D25	A64 := D26	A74 := D27
A45 := M4 - G54	A55 := M5 - G55	A65 := M6 - G56	A75 := M7 - G57
A46 := C54	A56 := C55	A66 := C56	A76 := C57
A47 := -M4 - G54	A57 := -M5 - G55	A67 := -M6 - G56	A77 := -M7 - G57



$A00 = -6.7$   $A10 = 9.1$   $A20 = -7.1$   $A30 = 8.7$   $A40 = -7.5$   $A50 = 8.3$   $A60 = -7.9$   $A70 = 7.9$   
 $A01 = 0.167$   $A11 = 0.167$   $A21 = 0.167$   $A31 = 0.167$   $A41 = 0.167$   $A51 = 0.167$   $A61 = 0.167$   $A71 = 0.167$   
 $A02 = 0.167$   $A12 = 0.167$   $A22 = 0.167$   $A32 = 0.167$   $A42 = 0.167$   $A52 = 0.167$   $A62 = 0.167$   $A72 = 0.167$   
 $A03 = 0.167$   $A13 = 0.167$   $A23 = 0.167$   $A33 = 0.167$   $A43 = 0.167$   $A53 = 0.167$   $A63 = 0.167$   $A73 = 0.167$   
 $A04 = 0.167$   $A14 = 0.167$   $A24 = 0.167$   $A34 = 0.167$   $A44 = 0.167$   $A54 = 0.167$   $A64 = 0.167$   $A74 = 0.167$   
 $A05 = 0.167$   $A15 = 0.167$   $A25 = 0.167$   $A35 = 0.167$   $A45 = 0.167$   $A55 = 0.167$   $A65 = 0.167$   $A75 = 0.167$   
 $A06 = 0.167$   $A16 = 0.167$   $A26 = 0.167$   $A36 = 0.167$   $A46 = 0.167$   $A56 = 0.167$   $A66 = 0.167$   $A76 = 0.167$   
 $A07 = 0.167$   $A17 = 0.167$   $A27 = 0.167$   $A37 = 0.167$   $A47 = 0.167$   $A57 = 0.167$   $A67 = 0.167$   $A77 = 0.167$

$G102 := A30 + A70$   $G302 := A10 + A50$   $G502 := A30 - A70$   
 $G202 := A20 + A40$   $G402 := A00 + A60$   $C102 := G102 + G302$   
 $C202 := G202 + G402$   $C502 := G102 - G302$   $C602 := G202 - G402$   $D102 := C102 + C202$   $D202 := C102 - C202$   
 $G112 := A31 + A71$   $G312 := A11 + A51$   $G512 := A31 - A71$   
 $G212 := A21 + A41$   $G412 := A01 + A61$   $C112 := G112 + G312$   
 $C212 := G212 + G412$   $C512 := G112 - G312$   $C612 := G212 - G412$   $D112 := C112 + C212$   $D212 := C112 - C212$   
 $G122 := A32 + A72$   $G322 := A12 + A52$   $G522 := A32 - A72$   
 $G222 := A22 + A42$   $G422 := A02 + A62$   $C122 := G122 + G322$   
 $C222 := G222 + G422$   $C522 := G122 - G322$   $C622 := G222 - G422$   $D122 := C122 + C222$   $D222 := C122 - C222$   
 $G132 := A33 + A73$   $G332 := A13 + A53$   $G532 := A33 - A73$   
 $G232 := A23 + A43$   $G432 := A03 + A63$   $C132 := G132 + G332$   
 $C232 := G232 + G432$   $C532 := G132 - G332$   $C632 := G232 - G432$   $D132 := C132 + C232$   $D232 := C132 - C232$   
 $G142 := A34 + A74$   $G342 := A14 + A54$   $G542 := A34 - A74$   
 $G242 := A24 + A44$   $G442 := A04 + A64$   $C142 := G142 + G342$   
 $C242 := G242 + G442$   $C542 := G142 - G342$   $C642 := G242 - G442$   $D142 := C142 + C242$   $D242 := C142 - C242$   
 $G152 := A35 + A75$   $G352 := A15 + A55$   $G552 := A35 - A75$   
 $G252 := A25 + A45$   $G452 := A05 + A65$   $C152 := G152 + G352$   
 $C252 := G252 + G452$   $C552 := G152 - G352$   $C652 := G252 - G452$   $D152 := C152 + C252$   $D252 := C152 - C252$   
 $G162 := A36 + A76$   $G362 := A16 + A56$   $G562 := A36 - A76$   
 $G262 := A26 + A46$   $G462 := A06 + A66$   $C162 := G162 + G362$   
 $C262 := G262 + G462$   $C562 := G162 - G362$   $C662 := G262 - G462$   $D162 := C162 + C262$   $D262 := C162 - C262$   
 $G172 := A37 + A77$   $G372 := A17 + A57$   $G572 := A37 - A77$   
 $G272 := A27 + A47$   $G472 := A07 + A67$   $C172 := G172 + G372$   
 $C272 := G272 + G472$   $C572 := G172 - G372$   $C672 := G272 - G472$   $D172 := C172 + C272$   $D272 := C172 - C272$

M02 := C602 T2	M12 := C612 T2	M22 := C622 T2	M32 := C632 T2
C00 := D102	C10 := D112	C20 := D122	C30 := D132
C01 := -M02 - G502	C11 := -M12 - G512	C21 := -M22 - G522	C31 := -M32 - G532
C02 := C502	C12 := C512	C22 := C522	C32 := C532
C03 := M02 - G502	C13 := M12 - G512	C23 := M22 - G522	C33 := M32 - G532
C04 := D202	C14 := D212	C24 := D222	C34 := D232
C05 := M02 - G502	C15 := M12 - G512	C25 := M22 - G522	C35 := M32 - G532
C06 := C502	C16 := C512	C26 := C522	C36 := C532
C07 := -M02 - G502	C17 := -M12 - G512	C27 := -M22 - G522	C37 := -M32 - G532

M42 := C642 T2	M52 := C652 T2	M62 := C662 T2	M72 := C672 T2
C40 := D142	C50 := D152	C60 := D162	C70 := D172
C41 := -M42 - G542	C51 := -M52 - G552	C61 := -M62 - G562	C71 := -M72 - G572
C42 := C542	C52 := C552	C62 := C562	C72 := C572
C43 := M42 - G542	C53 := M52 - G552	C63 := M62 - G562	C73 := M72 - G572
C44 := D242	C54 := D252	C64 := D262	C74 := D272
C45 := M42 - G542	C55 := M52 - G552	C65 := M62 - G562	C75 := M72 - G572
C46 := C542	C56 := C552	C66 := C562	C76 := C572
C47 := -M42 - G542	C57 := -M52 - G552	C67 := -M62 - G562	C77 := -M72 - G572

C00 = 4.8	C10 = 1.333	C20 = 1.333	C30 = 1.333	C40 = 1.333	C50 = 1.333	C60 = 1.333	C70 = 1.333
C01 = -0.8	C11 = 0	C21 = 0	C31 = 0	C41 = 0	C51 = 0	C61 = 0	C71 = 0
C02 = -0.8	C12 = 0	C22 = 0	C32 = 0	C42 = 0	C52 = 0	C62 = 0	C72 = 0
C03 = -0.8	C13 = 0	C23 = 0	C33 = 0	C43 = 0	C53 = 0	C63 = 0	C73 = 0
C04 = 63.2	C14 = 0	C24 = 0	C34 = 0	C44 = 0	C54 = 0	C64 = 0	C74 = 0
C05 = -0.8	C15 = 0	C25 = 0	C35 = 0	C45 = 0	C55 = 0	C65 = 0	C75 = 0
C06 = -0.8	C16 = 0	C26 = 0	C36 = 0	C46 = 0	C56 = 0	C66 = 0	C76 = 0
C07 = -0.8	C17 = 0	C27 = 0	C37 = 0	C47 = 0	C57 = 0	C67 = 0	C77 = 0

Cxx:=	{	C00	C01	C02	C03	C04	C05	C06	C07	{	4.8	-0.8	-0.8	-0.8	63.2	-0.8	-0.8	-0.8
		C10	C11	C12	C13	C14	C15	C16	C17		1.333	0	0	0	0	0	0	0
		C20	C21	C22	C23	C24	C25	C26	C27		1.333	0	0	0	0	0	0	0
		C30	C31	C32	C33	C34	C35	C36	C37		1.333	0	0	0	0	0	0	0
		C40	C41	C42	C43	C44	C45	C46	C47		1.333	0	0	0	0	0	0	0
		C50	C51	C52	C53	C54	C55	C56	C57		1.333	0	0	0	0	0	0	0
		C60	C61	C62	C63	C64	C65	C66	C67		1.333	0	0	0	0	0	0	0
		C70	C71	C72	C73	C74	C75	C76	C77		1.333	0	0	0	0	0	0	0

## Appendix 7

Example calculations of 2-Dimension Discrete Cosine Transform for 64 samples according to one embodiment of the present invention.

DCT\_2D\_GRAPH

8 Multiplies

163 Adds

One wired shift

### PROCESS EXAMPLE OF TWO DIMENSION 64-POINT DISCRETE COSINE TRANSFORM

#### DONE AS A SINGLE STEP SPECIAL CASE PROCESS

M := 3

N := 3

$F(x, y) := \cos(2 \cdot \pi \cdot M \cdot x) + \cos(2 \cdot \pi \cdot N \cdot y)$

T := 0.7071067812

$$S := \begin{pmatrix} F(.125, .125) & F(.25, .125) & F(.375, .125) & F(.5, .125) & F(.625, .125) & F(.75, .125) & F(.875, .125) & F(1, .125) \\ F(.125, .250) & F(.25, .250) & F(.375, .250) & F(.5, .250) & F(.625, .250) & F(.75, .250) & F(.875, .250) & F(1, .250) \\ F(.125, .375) & F(.25, .375) & F(.375, .375) & F(.5, .375) & F(.625, .375) & F(.75, .375) & F(.875, .375) & F(1, .375) \\ F(.125, .5) & F(.25, .5) & F(.375, .5) & F(.5, .5) & F(.625, .5) & F(.75, .5) & F(.875, .5) & F(1, .5) \\ F(.125, .625) & F(.25, .625) & F(.375, .625) & F(.5, .625) & F(.625, .625) & F(.75, .625) & F(.875, .625) & F(1, .625) \\ F(.125, .75) & F(.25, .75) & F(.375, .75) & F(.5, .75) & F(.625, .75) & F(.75, .75) & F(.875, .75) & F(1, .75) \\ F(.125, .875) & F(.25, .875) & F(.375, .875) & F(.5, .875) & F(.625, .875) & F(.75, .875) & F(.875, .875) & F(1, .875) \\ F(.125, 1) & F(.25, 1) & F(.375, 1) & F(.5, 1) & F(.625, 1) & F(.75, 1) & F(.875, 1) & F(1, 1) \end{pmatrix}$$

$$S = \begin{pmatrix} -1.414 & -0.707 & 0 & -1.707 & 0 & -0.707 & -1.414 & 0.293 \\ -0.707 & 0 & 0.707 & -1 & 0.707 & 0 & -0.707 & 1 \\ 0 & 0.707 & 1.414 & -0.293 & 1.414 & 0.707 & 0 & 1.707 \\ -1.707 & -1 & -0.293 & -2 & -0.293 & -1 & -1.707 & 0 \\ 0 & 0.707 & 1.414 & -0.293 & 1.414 & 0.707 & 0 & 1.707 \\ -0.707 & 0 & 0.707 & -1 & 0.707 & 0 & -0.707 & 1 \\ -1.414 & -0.707 & 0 & -1.707 & 0 & -0.707 & -1.414 & 0.293 \\ 0.293 & 1 & 1.707 & 0 & 1.707 & 1 & 0.293 & 2 \end{pmatrix}$$

S00 := S<sub>0,0</sub>    S01 := S<sub>0,1</sub>    S02 := S<sub>0,2</sub>    S03 := S<sub>0,3</sub>    S04 := S<sub>0,4</sub>    S05 := S<sub>0,5</sub>    S06 := S<sub>0,6</sub>    S07 := S<sub>0,7</sub>  
S10 := S<sub>1,0</sub>    S11 := S<sub>1,1</sub>    S12 := S<sub>1,2</sub>    S13 := S<sub>1,3</sub>    S14 := S<sub>1,4</sub>    S15 := S<sub>1,5</sub>    S16 := S<sub>1,6</sub>    S17 := S<sub>1,7</sub>  
S20 := S<sub>2,0</sub>    S21 := S<sub>2,1</sub>    S22 := S<sub>2,2</sub>    S23 := S<sub>2,3</sub>    S24 := S<sub>2,4</sub>    S25 := S<sub>2,5</sub>    S26 := S<sub>2,6</sub>    S27 := S<sub>2,7</sub>  
S30 := S<sub>3,0</sub>    S31 := S<sub>3,1</sub>    S32 := S<sub>3,2</sub>    S33 := S<sub>3,3</sub>    S34 := S<sub>3,4</sub>    S35 := S<sub>3,5</sub>    S36 := S<sub>3,6</sub>    S37 := S<sub>3,7</sub>  
S40 := S<sub>4,0</sub>    S41 := S<sub>4,1</sub>    S42 := S<sub>4,2</sub>    S43 := S<sub>4,3</sub>    S44 := S<sub>4,4</sub>    S45 := S<sub>4,5</sub>    S46 := S<sub>4,6</sub>    S47 := S<sub>4,7</sub>  
S50 := S<sub>5,0</sub>    S51 := S<sub>5,1</sub>    S52 := S<sub>5,2</sub>    S53 := S<sub>5,3</sub>    S54 := S<sub>5,4</sub>    S55 := S<sub>5,5</sub>    S56 := S<sub>5,6</sub>    S57 := S<sub>5,7</sub>  
S60 := S<sub>6,0</sub>    S61 := S<sub>6,1</sub>    S62 := S<sub>6,2</sub>    S63 := S<sub>6,3</sub>    S64 := S<sub>6,4</sub>    S65 := S<sub>6,5</sub>    S66 := S<sub>6,6</sub>    S67 := S<sub>6,7</sub>  
S70 := S<sub>7,0</sub>    S71 := S<sub>7,1</sub>    S72 := S<sub>7,2</sub>    S73 := S<sub>7,3</sub>    S74 := S<sub>7,4</sub>    S75 := S<sub>7,5</sub>    S76 := S<sub>7,6</sub>    S77 := S<sub>7,7</sub>

A00 := S00 + S06    A01 := S01 + S05    A02 := S02 + S04    A03 := S03 + S07    A04 := S03 - S07  
A10 := S10 + S16    A11 := S11 + S15    A12 := S12 + S14    A13 := S13 + S17    A14 := S13 - S17  
A20 := S20 + S26    A21 := S21 + S25    A22 := S22 + S24    A23 := S23 + S27    A24 := S23 - S27  
A30 := S30 + S36    A31 := S31 + S35    A32 := S32 + S34    A33 := S33 + S37    A34 := S33 - S37  
A40 := S40 + S46    A41 := S41 + S45    A42 := S42 + S44    A43 := S43 + S47    A44 := S43 - S47  
A50 := S50 + S56    A51 := S51 + S55    A52 := S52 + S54    A53 := S53 + S57    A54 := S53 - S57  
A60 := S60 + S66    A61 := S61 + S65    A62 := S62 + S64    A63 := S63 + S67    A64 := S63 - S67  
A70 := S70 + S76    A71 := S71 + S75    A72 := S72 + S74    A73 := S73 + S77    A74 := S73 - S77

B10:= A03 + A01	B50:= A03 - A01	B20:= A02 + A00	B60:= A02 - A00	B30:= A04 + A64
B11:= A13 + A11	B51:= A13 - A11	B21:= A12 + A10	B61:= A12 - A10	B31:= A14 + A54
B12:= A23 + A21	B52:= A23 - A21	B22:= A22 + A20	B62:= A22 - A20	B32:= A24 + A44
B13:= A33 + A31	B53:= A33 - A31	B23:= A32 + A30	B63:= A32 - A30	B33:= A34 + A74
B14:= A43 + A41	B54:= A43 - A41	B24:= A42 + A40	B64:= A42 - A40	
B15:= A53 + A51	B55:= A53 - A51	B25:= A52 + A50	B65:= A52 - A50	
B16:= A63 + A61	B56:= A63 - A61	B26:= A62 + A60	B66:= A62 - A60	
B17:= A73 + A71	B57:= A73 - A71	B27:= A72 + A70	B67:= A72 - A70	B37:= A34 - A74
C10:= B10 + B20	C20:= B10 - B20	C30:= B30 + B32	C31:= B30 - B32	C50:= B50 + B56
C11:= B11 + B21	C21:= B11 - B21	C32:= B31 + B33	C33:= B31 - B33	C51:= B51 + B55
C12:= B12 + B22	C22:= B12 - B22	C53:= B53 + B57	C57:= B53 - B57	C52:= B52 + B54
C13:= B13 + B23	C23:= B13 - B23	C63:= B63 + B67	C37:= B63 - B67	C60:= B60 + B66
C14:= B14 + B24	C24:= B14 - B24			C61:= B61 + B65
C15:= B15 + B25	C25:= B15 - B25			C62:= B62 + B64
C16:= B16 + B26	C26:= B16 - B26			
C17:= B17 + B27	C27:= B17 - B27			
D10:= C10 + C16	D20:= C20 + C26	D30:= C30 + C32		D32:= C30 - C32
D11:= C11 + C15	D21:= C21 + C25	D31:= C37 + C31		D37:= C37 - C31
D12:= C12 + C14	D22:= C22 + C24	D50:= C50 + C52		D52:= C50 - C52
D13:= C13 + C17	D23:= C23 + C27	D51:= C51 + C53		D53:= C51 - C53
D17:= C13 - C17	D27:= C23 - C27	D60:= C60 + C62		D62:= C60 - C62
		D61:= C61 + C63		D63:= C61 - C63
E10:= D10 + D12	E12:= D10 - D12	E21:= D21 + D23		E23:= D21 - D23
E11:= D11 + D13	E13:= D11 - D13	E32:= D50 + D51		E33:= D50 - D51
E20:= D20 + D22	E22:= D20 - D22	E30:= D60 + D61		E31:= D60 - D61
F10:= E10 + E11	F11:= E11 - E10	F20:= E20 + E21		F21:= E20 - E21
M1 := E12.T	M3 := E30.T	M5 := D31.T		M7 := D52.T
M2 := E22.T	M4 := E31.T	M6 := D37.T		M8 := D63.T
M9 := D62.5 Note: In a custom design this is only a wiring shift, not really a multiply.				
G22:= B37 + M9	G23:= B37 - M9			
H10:= M1 + D17	H20:= M1 - D17	H14:= M5 + G22		H24:= M5 - G22
H11:= M2 + D27	H21:= M2 - D27	H15:= M6 + G23		H25:= M6 - G23
H12:= M3 + D30	H22:= M3 - D30	H16:= M7 + C57		H26:= M7 - C57
H13:= M4 + D32	H23:= M4 - D32	H17:= M8 + C33		H27:= M8 - C33
Co00:= F10	Co20:= E32	Co40:= F20		Co60:= Co20
Co01:= H20	Co21:= H26	Co41:= H21		Co61:= Co21
Co02:= -E13	Co22:= -D53	Co42:= -E23		Co62:= Co22
Co03:= -H10	Co23:= -H16	Co43:= -H11		Co63:= Co23
Co04:= F11	Co24:= -E33	Co44:= -F21		Co64:= Co24
Co05:= Co03	Co25:= Co23	Co45:= Co43		Co65:= Co63
Co06:= Co02	Co26:= Co22	Co46:= Co42		Co66:= Co62
Co07:= Co01	Co27:= Co21	Co47:= Co41		Co67:= Co61
Co10:= -H12	Co30:= H22	Co50:= Co30		Co70:= Co10
Co11:= H15	Co31:= -H24	Co51:= Co31		Co71:= Co11
Co12:= H17	Co32:= -H27	Co52:= Co32		Co72:= Co12
Co13:= H14	Co33:= -H25	Co53:= Co33		Co73:= Co13
Co14:= H13	Co34:= -H23	Co54:= Co34		Co74:= Co14
Co15:= Co13	Co35:= Co33	Co55:= Co53		Co75:= Co73
Co16:= Co12	Co36:= Co32	Co56:= Co52		Co76:= Co72
Co17:= Co11	Co37:= Co31	Co57:= Co51		Co77:= Co71

# OUTPUT SHEET FOR 2-DIMENSION DISCRETE COSINE TRANSFORM

M:=2      N:=3

$$S = \begin{pmatrix} -1.414 & -0.707 & 0 & -1.707 & 0 & -0.707 & -1.414 & 0.293 \\ -0.707 & 0 & 0.707 & -1 & 0.707 & 0 & -0.707 & 1 \\ 0 & 0.707 & 1.414 & -0.293 & 1.414 & 0.707 & 0 & 1.707 \\ -1.707 & -1 & -0.293 & -2 & -0.293 & -1 & -1.707 & 0 \\ 0 & 0.707 & 1.414 & -0.293 & 1.414 & 0.707 & 0 & 1.707 \\ -0.707 & 0 & 0.707 & -1 & 0.707 & 0 & -0.707 & 1 \\ -1.414 & -0.707 & 0 & -1.707 & 0 & -0.707 & -1.414 & 0.293 \\ 0.293 & 1 & 1.707 & 0 & 1.707 & 1 & 0.293 & 2 \end{pmatrix}$$

ARRAY OF INPUT VALUES

$$C_{xx} := \begin{pmatrix} Co00 & Co01 & Co02 & Co03 & Co04 & Co05 & Co06 & Co07 \\ Co10 & Co11 & Co12 & Co13 & Co14 & Co15 & Co16 & Co17 \\ Co20 & Co21 & Co22 & Co23 & Co24 & Co25 & Co26 & Co27 \\ Co30 & Co31 & Co32 & Co33 & Co34 & Co35 & Co36 & Co37 \\ Co40 & Co41 & Co42 & Co43 & Co44 & Co45 & Co46 & Co47 \\ Co50 & Co51 & Co52 & Co53 & Co54 & Co55 & Co56 & Co57 \\ Co60 & Co61 & Co62 & Co63 & Co64 & Co65 & Co66 & Co67 \\ Co70 & Co71 & Co72 & Co73 & Co74 & Co75 & Co76 & Co77 \end{pmatrix}$$

MAP NAMES TO LOCATIONS

$$C_{xx} = \begin{pmatrix} 0 & 0 & 0 & 32 & 0 & 32 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

ARRAY OF OUTPUT VALUES

DCT\_2D\_GRAPH

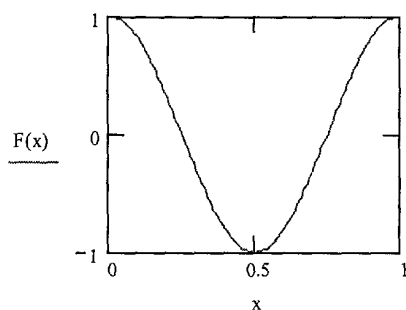
## Appendix 8

Example calculations of Discrete Cosine Transform for eight samples according to one embodiment of the present invention.

### Super Fast Discrete Cosine Transform

Let  $x$  range from  $1/8$  to  $1$  in 8 steps. Let  $F(x)$  be sampled at the eight points.

$$x := 0, .01.. 1 \quad F(x) := \cos(1 \cdot 2 \cdot \pi \cdot x)$$



$$S := \begin{pmatrix} F(.125) \\ F(.250) \\ F(.375) \\ F(.5) \\ F(.625) \\ F(.75) \\ F(.875) \\ F(1) \end{pmatrix} \quad S = \begin{pmatrix} 0.707 \\ 0 \\ -0.707 \\ -1 \\ -0.707 \\ 0 \\ 0.707 \\ 1 \end{pmatrix}$$

$$T2 := 0.7071067812$$

$$M1 := (S_0 - S_2 - S_4 + S_6) \cdot T2 \quad \text{Multiply} \quad M1 = 2$$

$$A0 := (S_0 + S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7) \quad A0 = 0$$

$$A1 := M1 - S_3 + S_7 \quad A1 = 4$$

$$A2 := -S_1 + S_3 - S_5 + S_7 \quad A2 = 0$$

$$A3 := -M1 - S_3 + S_7 \quad A3 = 0$$

$$A4 := -S_0 + S_1 - S_2 + S_3 - S_4 + S_5 - S_6 + S_7 \quad A4 = 0$$

$$A5 := -M1 - S_3 + S_7 \quad A5 = 0$$

$$A6 := -S_1 + S_3 - S_5 + S_7 \quad A6 = 0$$

$$A7 := M1 - S_3 + S_7 \quad A7 = 4$$

$$FT := \begin{pmatrix} A0 & A1 & A2 & A3 \\ A4 & A5 & A6 & A7 \end{pmatrix} \quad FT = \begin{pmatrix} 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

## Appendix 9

Description of an example process for calculating Fourier Transform for eight samples according to one embodiment of the present invention.

### Process Example of SFT

Process Example of SFT						
Let x range from 1/8 to 1 in 8 steps. Let F(x) be sampled at the eight points (S).						F(.125)
F(x) := cos(4·2·π ·x)						F(.250)
T2 := 0.7071067812						F(.375)
						F(.5)
G4 := S <sub>0</sub>	G6 := S <sub>0</sub>	1st sample arrival. Load to two memory locations.				S := F(.625)
G3 := S <sub>1</sub>	G7 := S <sub>1</sub>	2nd sample arrival. Load to two memory locations.				F(.75)
G2 := S <sub>2</sub>	G8 := S <sub>2</sub>	3rd sample arrival. Load to two memory locations.				F(.875)
G1 := S <sub>3</sub>	G5 := S <sub>3</sub>	4th sample arrival. Load to two memory locations.				F(1)
G2 := G2 + S <sub>4</sub>	G8 := G8 - S <sub>4</sub>	5th sample arrival. Add-accumulate two locations.				A-A=2
G3 := G3 + S <sub>5</sub>	G7 := G7 - S <sub>5</sub>	6th sample arrival. Add-accumulate two locations.				A-A=4
G4 := G4 + S <sub>6</sub>	G6 := G6 - S <sub>6</sub>	7th sample arrival. Add-accumulate two locations.				A-A=6
TEMP := G2 - G4	G4 := G2 + G4	G4 is C2	G2 := TEMP	G2 is C6	A-A=8	
TEMP := G6 + G8	G6 := G6 - G8	G6 is C3	G8 := TEMP	G8 is C4	A-A=10	
TEMP := G8·T2						Mult=1
First Multiply is stored in TEMP						
G8 := TEMP - G7		G7 := TEMP + G7		A-A=12		
B1 := G7	G7 HOLDS B1		B1 = 0			
B2 := G6	G6 HOLDS B2		B2 = 0			
B3 := G8	G8 HOLDS B3		B3 = 0			
G1 := G1 + S <sub>7</sub>	G5 := G5 + S <sub>7</sub>	8th sample arrival. Add-accumulate two locations.				A-A=14
TEMP := G1 + G3	G1 := G1 - G3	G1 is C5	G3 := TEMP	G3 is C1	A-A=16	
TEMP := G3 + G4	G3 := G3 - G4	G3 is D2	G4 := TEMP	G4 is D1	A-A=18	
A0 := G4	G4 HOLDS A0		$\frac{A0}{2} = 0$			
A2 := G1	G1 HOLDS A2		A2 = 0			
A4 := G3	G3 HOLDS A4		$\frac{A4}{2} = 4$			
TEMP := G2·T2						Mult=2
Second Multiply is stored in TEMP						
G2 := G5 - TEMP		G5 := G5 + TEMP		A-A=20		
A1 := G2	G2 HOLDS A1		A1 = 0			
A3 := G5	G5 HOLDS A3		A3 = 0			
DFT :=						
$\begin{bmatrix} A1 & A2 & A3 & \frac{A4}{2} \\ B1 & B2 & B3 & \frac{A0}{2} \end{bmatrix}$						
DFT =						
$\begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$						
File: SFT_Ex.doc						

## Appendix 10

Description of an example process for calculating Discrete Cosine Transform for eight samples according to one embodiment of the present invention.

### Process Example of SDCT

<p>Let x range from 1/8 to 1 in 8 steps. Let F(x) be sampled at the eight points (S).</p> <p><math>F(x) := \cos(3 \cdot 2 \cdot \pi \cdot x)</math>                      <math>T2 := 0.7071067812</math></p>		$S := \begin{bmatrix} F(.125) \\ F(.250) \\ F(.375) \\ F(.5) \\ F(.625) \\ F(.75) \\ F(.875) \\ F(1) \end{bmatrix}$
<p>G4 := S<sub>0</sub>                      1st sample arrival. Load to memory location.</p> <p>G3 := S<sub>1</sub>                      2nd sample arrival. Load to memory location.</p> <p>G2 := S<sub>2</sub>                      3rd sample arrival. Load to memory location.</p> <p>G1 := S<sub>3</sub>            G5 := S<sub>3</sub>            4th sample arrival. Load to two memory locations.</p> <p>G2 := G2 + S<sub>4</sub>                      5th sample arrival. Add-accumulate.                      A-A=1</p> <p>G3 := G3 + S<sub>5</sub>                      6th sample arrival. Add-accumulate.                      A-A=2</p> <p>G4 := G4 + S<sub>6</sub>                      7th sample arrival. Add-accumulate.                      A-A=3</p> <p>G1 := G1 + S<sub>7</sub>    G5 := G5 - S<sub>7</sub>    8th sample arrival. Add-accumulate two locations.    A-A=5</p> <p>TEMP := G1 + G3            G1 := G1 - G3    G1 is C5    G3 := TEMP    G3 is C1    A-A=7</p> <p>TEMP := G2 - G4            G4 := G2 + G4    G4 is C2    G2 := TEMP    G2 is C6    A-A=9</p> <p>TEMP := G3 + G4            G3 := G3 - G4    G3 is D2    G4 := TEMP    G4 is D1    A-A=11</p>		
<p>A0 := G4                      G4 HOLDS A0                      A0 = 0</p> <p>A2 := G1                      G1 HOLDS A2                      A2 = 0</p> <p>A4 := G3                      G3 HOLDS A4                      A4 = 0</p> <p>A6 := G1                      G1 HOLDS A6                      A6 = 0</p>		
<p>TEMP := G2 · T2                      Multiply is stored in TEMP                      Mult=1</p> <p>G2 := -TEMP - G5    G5 := TEMP - G5                      A-A=13</p>		
<p>A1 := G2                      G2 HOLDS A1                      A1 = 0</p> <p>A3 := G5                      G5 HOLDS A3                      A3 = 4</p> <p>A5 := G5                      G5 HOLDS A3                      A5 = 4</p> <p>A7 := G2                      G2 HOLDS A1                      A7 = 0</p>		

$$DFT := \begin{bmatrix} A0 & A1 & A2 & A3 \\ A4 & A5 & A6 & A7 \end{bmatrix}$$

$$DFT = \begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 4 & 0 & 0 \end{bmatrix}$$

File: FT\_SDCT.DOC



## Appendix 11

TABLE 1 1-Dimension 16-Sample Discrete Fourier Transform

Co#	Formula	T2A	T2B	T2C	T2D	T3A	T3B	T3C	T3D	T4A	T4B	T4C	T4D	T5A	T5B	T5C	T5D
A1	S1+S15																
A2	S2+S14																
A3	S3+S13																
A4	S4+S12																
A5	S5+S11																
A6	S6+S10																
A7	S7+S9																
A8	S8+S16																
A9	S8-S16																
B1	A1+A7																
B2	A2+A6																
B3	A3+A5																
B4	A4+A8																
B5	A1-A7																
B6	A2-A6																
B7	A3-A5																
B8	A4-A8																
C1	B1+B3																
C2	B2+B4																
C3	B1-B3																
C4	B2-B4																
D1	C1+C2																
D2	C1-C2																
T1	0																
T2	1																
T3	0.9238795325																
T4	0.7071067814																
T5	0.3826834324																
Co1	D1				D1												
Co2	-A9+B5*T3+B6*T4+B7*T5	-A9					B5				B6				B7		
Co3	-B8+C3*T4		-B8									C3					
Co4	-A9-B7*T3-B6*T4+B5*T5	-A9					-B7				-B6				B5		
Co5	-C4			-C4													
Co6	-A9+B7*T3-B6*T4-B5*T5	-A9					B7				B6				-B5		
Co7	-B8-D3*T4		-B8									-C3					
Co8	-A9-B5*T3-B6*T4-B7*T5	-A9					-B5				-B6				-B7		

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059	2060	2061	2062	2063	2064	2065	2066	2067	2068	2069	2070	2071	2072	2073	2074	2075	2076	2077	2078	2079	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	2100
1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059	2060	2061	2062	2063	2064	2065	2066	2067	2068	2069	2070	2071	2072	2073	2074	2075	2076	2077	2078	2079	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	2100	

Co#	Formula	T2A	T2B	T2C	T2D	T3A	T3B	T3C	T3D	T4A	T4B	T4C	T4D	T5A	T5B	T5C	T5D
Co9	-D2				-D2												
Co10	-A9-B5*T3-B6*T4-B7*T5	-A9				-B5					-B6				-B7		
Co11	-B8-D3*T4		-B8									-C3					
Co12	-A9+B7*T3-B6*T4-B5*T5	-A9				B7					B6				-B5		
Co13	-C4			-C4													
Co14	-A9-B7*T3-B6*T4+B5*T5	-A9				-B7					-B6				B5		
Co15	-B8+C3*T4		-B8									C3					
Co16	-A9+B5*T3+B6*T4+B7*T5	-A9				B5					B6				B7		

TABLE 2. 1-Dimension 16-Sample Discrete Cosine Transform Illustrated

Co	Formula	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16
Identifier	Formula																
A1	S1+S15	+														+	
A2	S2+S14		+												+		
A3	S3+S13			+										+			
A4	S4+S12				+								+				
A5	S5+S11					+						+					
A6	S6+S10						+					+					
A7	S7+S9							+		+							
A8	S8+S16								+							+	
A9	S8-S16								+								-
B1	A1+A7	+						+		+						+	
B2	A2+A6		+				+				+				+		
B3	A3+A5			+		+						*		+			
B4	A4+A8				+				+				+			+	
B5	A1-A7	+						-		-						+	
B6	A2-A6		+				-			-					+		
B7	A3-A5			+		-						-		+			
B8	A4-A8				+				-				+				-
C1	B1+B3	+		+		+		+		+		+		+		+	
C2	B2+B4		+		+		+		+		+		+		+		+
C3	B1-B3	+		-		-		+		+		-		-		+	
C4	B2-B4		+		-		+		-		+		-		*		-
D1	C1+C2	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
D2	C1-C2	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
Token	Value																
T1	0																
T2	1																
T3	0.9238795325																
T4	0.7071067814																
T5	0.3826834324																
Multiply ID	Formula																
M1	B5*T3	T3						-T3		-T3						T3	
M2	B5*T5	T5						-T5		-T5						T5	
M3	B7*T3			T3		-T3						-T3		T3			
M4	B7*T5			T5		-T5						-T5		T5			
M5	B6*T4		T4					-T4				-T4			T4		
M6	C3*T4	T4		-T4		-T4		T4		T4		-T4		-T4		T4	

Co	Formula	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16
Identifier	Formula																
E1	-A9+M5		T4				-T4		-1		-T4				T4		1
E2	-A9-M5		-T4				T4		-1		T4				-T4		1
E3	M1+M4	T3		T5		-T5		-T3		-T3		-T5		T3		T3	
E4	M2-M3	T5		-T3		T3		-T5		-T5		T3		-T3		T5	
E5	-M6-B8	-T4		T4	-1	T4		-T4	1	-T4		T4	-1	T4		-T4	1
E6	M6-B8	T4		-T4	-1	-T4		T4	1	T4		-T4	-1	-T4		T4	1
Coefficient	Formula	T2A	T2B	T2C	T2D	T3A	T3B	T3C	T3D	T4A	T4B	T4C	T4D	T5A	T5B	T5C	T5D
Co1	D1				D1												
Co2	E1+E3	-A9					M1				M5				M4		
Co3	E5		-B8									-M6					
Co4	E2+E4	-A9					-M3				-M5				M2		
Co5	-C4			-C4													
Co6	E2-E4	-A9					M3				-M5				-M2		
Co7	E6		-B8									M6					
Co8	E1-E3	-A9					-M1				M5				-M4		
Co9	-D2			-D2													
Co10	E1-E3	-A9					-M1				M5				-M4		
Co11	E6		-B8									M6					
Co12	E2-E4	-A9					M3				-M5				-M2		
Co13	-C4			-C4													
Co14	E2+E4	-A9					-M3				-M5				M2		
Co15	E5		-B8									-M6					
Co16	E1+E3	-A9					M1				M5				M4		